

**Suppressing lepton flavor violation in a soft-wall extra dimension**Michael Atkins<sup>\*</sup> and Stephan J. Huber<sup>†</sup>*Department of Physics and Astronomy, University of Sussex, Brighton BN1 9QH, United Kingdom*

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A soft-wall warped extra dimension allows one to relax the tight constraints imposed by electroweak data in conventional Randall-Sundrum models. We investigate a setup, where the lepton flavor structure of the standard model is realized by split fermion locations. Bulk fermions with general locations are not analytically tractable in a soft-wall background, so we follow a numerical approach to perform the Kaluza-Klein reduction. Lepton flavor violation is induced by the exchange of Kaluza-Klein gauge bosons. We find that rates for processes such as muon-electron conversion are significantly reduced compared to hard-wall models, allowing for a Kaluza-Klein scale as low as 2 TeV. Accommodating small neutrino masses forces one to introduce a large hierarchy of scales into the model, making pressing the question of a suitable stabilization mechanism.

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**I. INTRODUCTION**

Over the last ten years, there has been a large increase in the study of extra dimensional models following the realization that they could help explain some of the unresolved problems in the standard model (SM). In 1999, Randall and Sundrum showed that a warped extra dimension could offer a geometric solution to the gauge hierarchy problem [1]. In the original Randall-Sundrum (RS) model, the fifth dimension consists of a slice of anti-de Sitter (AdS) space bounded by ultraviolet (UV) and infrared (IR) branes. The warped space produces an exponential difference in energy scales between the two branes which solves the hierarchy problem. Matter fields were originally confined to the IR brane; however, it was soon realized that by allowing fermions to propagate in the extra dimension, the SM fermion mass hierarchy can be explained. By varying the location of the fermion wave functions in the fifth dimension, the full scale of fermion masses from neutrinos to the top quark can be generated using only order unity parameters [2–4]. This setup also contains a built in mechanism suppressing unobserved flavor changing processes that result from couplings between SM fermions and excited gauge bosons which appear in the model [3,5,6].

Further interest in warped extra dimensions was generated by the AdS/CFT conjecture, when it was realized that the RS scenario is holographically dual to strongly coupled four-dimensional (4D) field theories [7–9]. It was in this context, studying AdS/QCD models, that the idea of a soft wall was first introduced [10]. The soft wall is realized by removing the IR brane so the extra dimension extends to infinity, and by replacing it with a smoothly varying space-time cutoff. The original AdS/QCD motivation for this was to more faithfully reproduce the linear Regge-like mass

squared spectrum of excited mesons as opposed to the usual quadratic spectrum found in hard-wall RS models.

Inspired by the possibility of qualitatively different phenomenology, the soft-wall scenario was subsequently applied to modelling electroweak physics [11,12]. These models successfully showed that a soft-wall extra dimension is generally less constrained by electroweak precision observables than its hard-wall counterpart, typically allowing Kaluza-Klein (KK) modes with masses of a few TeV. An important issue is related to the stability of a soft-wall setup, which is an open question in the models discussed in Refs. [11,12]. Such a mechanism was suggested in Ref. [13], promising the soft-wall extra dimension to equally well resolve the gauge hierarchy problem.

With the removal of the hard-wall brane the standard model matter fields must necessarily propagate in the bulk. Graviton fluctuations and gauge fields were successfully analyzed in this background, but it was found that fermions presented particular technical difficulties and only a simplified single generational model was developed. Later studies of fermions in a soft-wall extra dimension have developed solutions to the fermion problem [14–16] and have considered the experimental constraints imposed by the electroweak observables. However, the fermion flavor pattern of the SM has not been considered in much detail, in particular, with respect to the generation of neutrino masses and the experimental bounds on lepton flavor violation.

In this paper we present a numerical solution to analyze a single generation of fermions in the soft-wall extra dimension. We extend this solution to three generations by treating flavor mixings as perturbations to the original solutions, and apply it to the lepton sector of the SM. We construct a setup, where the lepton flavor pattern is accommodated by flavor dependent localizations. It is shown that in order generate small Dirac neutrino masses by this mechanism we need to introduce a hierarchy of scales of order  $10^{15}$  into the model, making crucial the issue of a

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suitable stabilization mechanism. We finally carry out an analysis of the constraints coming from various lepton flavor violating processes, averaging over random order unity Yukawa couplings, and find that models with only a modest hierarchy of scales are relatively mildly constrained; whereas, the model with a large hierarchy allowing sub-eV neutrino masses lies well within current experimental constraints, even for a KK scale<sup>1</sup> of 2 TeV. In the latter, flavor violation is considerably suppressed relative to its hard-wall counterparts, such as the ones analyzed in [5,17], and the range of masses lies in the reach of the LHC experiment.

At this stage we do not try to accommodate the flavor structure of the quark sector, which should be possible in a similar way. Also, we reproduce the neutrino masses and mixings only at the qualitative level, which is sufficient to estimate the rates of lepton flavor violation.

## II. BULK FIELDS IN A SOFT-WALL EXTRA DIMENSION

Our conventions follow most closely those laid down in Refs. [12,16]. The five-dimensional (5D) spacetime has metric

$$ds^2 = e^{-2A(y)} \eta_{MN} dx^M dx^N, \quad (1)$$

where  $y$  represents the extra spatial dimension and  $\eta_{MN} = \text{diag}(+, -, -, -, -)$ . We take a pure AdS metric,  $A(y) = \log ky$ , where  $k$  is the AdS curvature scale. There is no IR brane, the extra dimension extends to infinity, and the soft wall is introduced via a dilaton field  $\Phi$  with the action describing gauge and matter fields given by

$$S = \int d^4x \int_{y_0}^{\infty} dy \sqrt{g} e^{-\Phi} \mathcal{L}. \quad (2)$$

Here  $y_0 = 1/k$  is the location of the UV brane. The dilaton field is taken to have the following power law behavior

$$\Phi(y) = (\mu y)^2. \quad (3)$$

The dimensionful parameter  $\mu$  will set the mass scale of the lightest KK excitations. The behavior of other powers has been discussed in detail in Ref. [14]. It is shown in Ref. [12] that an appropriate form for the Higgs vacuum expectation value (VEV) in such a background is given by

$$h(y) = \eta k^{3/2} \mu^2 y^2, \quad (4)$$

where  $\eta$  is a dimensionless  $\mathcal{O}(1)$  coefficient.

### A. Massive gauge fields

In Refs. [12,16] only massless gauge fields are considered, as the gauge couplings being considered are assumed to be between quarks and massless gluons. The flavor

changing neutral currents (FCNC) we will be considering here are mediated by the massive  $Z$  boson, so we first develop the solutions for such a gauge field. A massive gauge field propagating in the bulk has the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{MN} F^{MN} + \frac{1}{2} M_A^2 A_M A^M.$$

The mass term  $M_A^2$  for the weak gauge bosons arises from spontaneous symmetry breaking and with the Higgs VEV as given in (4), we have  $M_A^2 = \frac{1}{2} g_5^2 h(y)^2$ . Varying the action (2), we obtain the equation of motion

$$\frac{1}{\sqrt{g}} \partial_M (\sqrt{g} g^{MN} g^{RS} F_{NS}) - g^{MN} g^{RS} F_{NS} \partial_R \Phi - M_A^2 G^{MN} A_M = 0.$$

Imposing the gauge  $A_y = 0$ , inserting the Kaluza-Klein (KK) reduction,

$$A_\mu(x, y) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x) f_A^{(n)}(y), \quad (5)$$

and requiring the  $A_\mu^{(n)}(x)$  to be mass eigenstates, we find the  $f_A^{(n)}$  have to satisfy

$$\left( \partial_y^2 - \left( \frac{1}{y} + \Phi' \right) \partial_y - \frac{1}{(ky)^2} M_A^2 + m_n^2 \right) f_A^{(n)}(y) = 0 \quad (6)$$

and are canonically normalized by

$$\int_{y_0}^{\infty} dy e^{-(A+\Phi)} f_A^{(m)}(y) f_A^{(n)}(y) = \delta^{mn}.$$

A complicated analytic solution of the above equation of motion was developed in [14], but for our purposes, we find it more convenient to solve the equation of motion numerically. We apply Neumann boundary conditions to the wave functions at  $y_0$  and vary  $\eta g_5$  in order to find a normalizable solution with the appropriate 4D zero mode mass of  $m_0 \simeq 91$  GeV for the  $Z$  boson.

It is interesting to note that unlike in hard-wall models, the profile of a massive gauge boson is independent of the curvature scale  $k$ . This can easily be seen by looking at the equation of motion (6) where  $k$  only appears in the term involving the 5D mass  $M_A$  which is the term that must be varied in order to generate the correct zero mode mass. The profile of the first few KK modes of the  $Z$  boson are plotted in Fig. 1. In Fig. 1(a) the zero mode and the first two KK modes are plotted with respect to a flat metric. Figure 1(b) shows the profile of the zero mode in the form that it couples to the fermions (see later). The UV behavior of the zero mode is less flat than in hard-wall models and could possibly lead to large violations of universality of gauge couplings once fermions reside at different locations in the extra dimension. Note also that as is the case for massless gauge bosons [12,16], the higher KK modes become more and more IR localized, a fact that will be

<sup>1</sup>When referring to the soft-wall model, we take the KK scale to be the mass of the first KK mode of the  $Z$  boson.

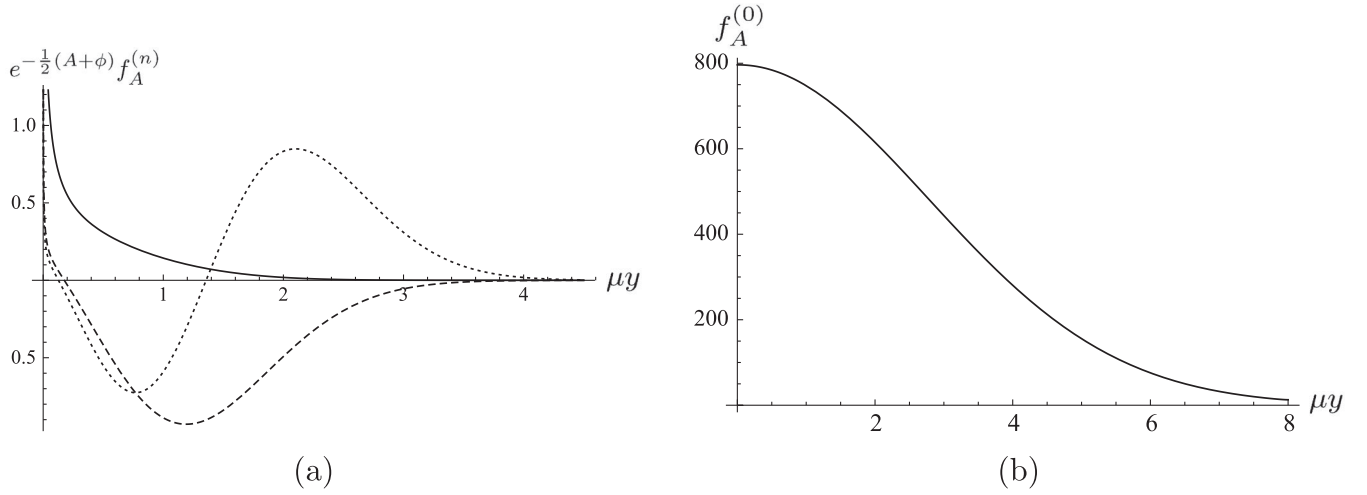


FIG. 1. Z boson profiles for  $k = 10^7$  TeV and  $\mu = 1$  TeV. (a) The zero mode (solid), first (dashed), and second (dotted) KK modes plotted with respect to a flat metric. (b) The zero mode profile as it couples to fermions.

important when considering couplings between higher gauge modes and fermions.

The KK spectrum for the Z boson for different values of  $\mu$  with  $k = 10^7$  TeV is

$\mu = \frac{1}{2}$ TeV: $m_0 = 0.091$ TeV,	$m_1 = 1.3$ TeV,	$m_2 = 1.8$ TeV,
$\mu = 1$ TeV: $m_0 = 0.091$ TeV,	$m_1 = 2.2$ TeV,	$m_2 = 3.0$ TeV,
$\mu = 2$ TeV: $m_0 = 0.091$ TeV,	$m_1 = 4.1$ TeV,	$m_2 = 5.8$ TeV,
$\mu = 4$ TeV: $m_0 = 0.091$ TeV,	$m_1 = 8.2$ TeV,	$m_2 = 12$ TeV.

Note that the mass of the first KK mode  $m_1 \sim 2\mu$ , and hence the KK scale,  $M_{\text{KK}}$ , scales with  $\mu$ . Higher modes follow a Regge-like spectrum  $m_n^2 \sim n$ .

## B. Fermions

We consider 5D Dirac spinors  $\Psi_L$  and  $\Psi_R$  which are components of doublets and singlets under  $SU(2)_L$ , respectively. Note that  $L$  and  $R$  do not denote chiralities, but are related to the charges under  $SU(2)_L$ . The chiral projections of these spinors are  $\Psi_{L\pm} = \frac{1}{2}(1 \mp \gamma^5)\Psi_L$ , same for  $\Psi_R$ . The action for two free fermions in the bulk is

$$\begin{aligned}
 S = & \int d^4x \int_{y_0}^{\infty} dy \sqrt{g} e^{-\Phi} \\
 & \times \left[ \frac{1}{2} (\bar{\Psi}_L i e_A^M \gamma^A D_M \Psi_L - D_M \bar{\Psi}_L i e_A^M \gamma^A \Psi_L) \right. \\
 & - M_L \bar{\Psi}_L \Psi_L + \frac{1}{2} (\bar{\Psi}_R i e_A^M \gamma^A D_M \Psi_R \\
 & \left. - D_M \bar{\Psi}_R i e_A^M \gamma^A \Psi_R) - M_R \bar{\Psi}_R \Psi_R \right].
 \end{aligned}$$

The fünfbein and spin connection for the metric (1) are  $e_A^M = e^{A(y)} \delta_A^M$  and  $\omega_M = (-\frac{A'}{2} \gamma_\mu \gamma^5, 0)$  and the covariant derivative is then  $D_M = \partial_M + \omega_M$ .  $M_{L,R}$  are the 5D Dirac masses related to  $\Psi_{L,R}$ .

The difficulty of placing uncoupled fermions in the soft-wall background is well documented in Ref. [12], all solutions suffer from divergent gauge couplings for high enough KK modes. The underlying problems stem from the noncompact nature of the extra dimension, and it is shown that in order to find workable normalizable solutions, the Yukawa couplings between fermions and the Higgs must be taken into account. An alternative approach to introducing Yukawa couplings was presented in Ref. [15] where a  $y$  dependent Dirac mass term is introduced, somewhat like the  $y$  dependent bulk mass arising from the Higgs VEV in the case of the massive gauge boson above. Here, however, we will stick to constant Dirac mass terms  $M_L$  and  $M_R$  and introduce Yukawa couplings into the following action:

$$\begin{aligned}
 S_{\text{Yuk}} = & - \int d^4x \int_{y_0}^{\infty} dy \sqrt{g} e^{-\Phi} \frac{\lambda_5}{\sqrt{k}} [\bar{\Psi}_L(x, y) h(y) \Psi_R(x, y) \\
 & + \bar{\Psi}_R(x, y) h(y) \Psi_L(x, y)].
 \end{aligned}$$

Defining  $\Psi_{L,R} = e^{2A+\Phi/2} \psi_{L,R}$  and  $m(y) = \frac{\lambda_5}{\sqrt{k}} h(y)$ , the equations of motion are

$$\begin{aligned}
 i \gamma^\mu \partial_\mu \psi_{L,R\pm} \pm \partial_y \psi_{L,R\mp} - e^{-A} M_{L,R} \psi_{L,R\mp} \\
 - e^{-A} m(y) \psi_{R,L\mp} = 0.
 \end{aligned}$$

Using the KK reduction

$$\psi_{L,R\pm}(x, y) = \sum_{n=0}^{\infty} \psi_{\pm}^{(n)}(x) f_{L,R\pm}^{(n)}(y),$$

and requiring the  $\psi_{\pm}^{(n)}(x)$  to be mass eigenstates, the  $f^{(n)}$ s will be given by

$$\pm \partial_y \begin{pmatrix} f_{L\pm}^{(n)} \\ f_{R\pm}^{(n)} \end{pmatrix} + e^{-A} \begin{pmatrix} M_L & m(y) \\ m(y) & M_R \end{pmatrix} \begin{pmatrix} f_{L\pm}^{(n)} \\ f_{R\pm}^{(n)} \end{pmatrix} = m_n \begin{pmatrix} f_{L\pm}^{(n)} \\ f_{R\pm}^{(n)} \end{pmatrix}. \quad (7)$$

The  $\psi_{\pm}^{(n)}$  will be canonically normalized by

$$\int_{z_0}^{\infty} dy (f_{L\pm}^{(m)} f_{L\pm}^{(n)} + f_{R\pm}^{(m)} f_{R\pm}^{(n)}) = \delta_{mn}.$$

We impose Dirichlet boundary conditions at  $y_0$  for  $f_{L-}^{(n)}$  and  $f_{R+}^{(n)}$  in order to obtain a chiral 4D theory.

Analytic solutions of (7) are only possible for a small set of Dirac mass terms, namely  $M_L = M_R$  and  $M_L + M_R \pm k = 0$  [16]. As in hard-wall models, the Dirac mass parameters  $M_{L,R}$  dictate how the fermion is localized in the extra dimension and it is convenient to parametrise these in terms of the AdS curvature,  $M_{L,R} = c_{L,R}k$ . Unfortunately, the sets of parameters for which analytic solutions are available do not explore the full geography of possible mass parameters and, as we shall see, may lead to situations with unacceptably large rates of flavor violation. Ideally, we would like to solve (7) for any set of Dirac masses and this requires a numerical approach.

The numerical solution we have developed involves a shooting type method. In order for the solutions to be normalizable, they must not diverge in the IR and this only occurs for the correct choice of  $m_n$ , thus generating the KK spectrum. We choose a suitably large distance  $L$  into the IR and solve the equations of motion subject to the UV boundary conditions and a starting choice for  $m_n$ . We then iterate the solution using Newton's method in order to find a value for  $m_n$  such that two of the solutions e.g.  $f_{L+}^{(n)}$  and  $f_{R-}^{(n)}$  converge to zero at  $y = L$ . The equations of motion then automatically ensure that the other two solutions will also converge to zero for large  $y$ . Our solution has the advantage that it seems to be quite capable of finding solutions even for large values of the AdS curvature scale  $k$ , however it is not so suited to finding solutions for multiple generations of fermions as is done in Ref. [16].

### III. LEPTONS

#### A. General considerations

Because of the presence of the extra KK states in extra dimensional models, couplings between SM particles and their KK excitations can potentially lead to conflict with experimental observations. In the SM, a tight set of constraints comes from the experimental bounds on FCNC.

In the hard-wall Randall-Sundrum model these processes have been investigated and are shown to occur at rates that are dependent on the fermion locations. However, there are certain choices of fermion locations which provide almost universal gauge couplings, and these almost universal gauge couplings are the source of the so called RS Glashow-Iliopoulos-Maiani mechanism which suppresses FCNC [3,5,6].

It was found in Ref. [12] that for fermions in the soft-wall background, the analytic solution with  $M_L = M_R$  can produce a large hierarchy of masses but only one of the fermion pair ( $\psi_+$ ,  $\psi_-$ ) could reside in an area of universal gauge couplings and it was thus assumed that dangerous rates of FCNC would be generated in such a situation. In fact, this is one of the main motivations for finding a numerical solution to the fermion equations of motion, in the hope that one would be able to find fermion locations which can give a large mass hierarchy and yet simultaneously reside in an area of universal gauge couplings.

The gauge interaction between bulk gauge bosons and fermions is given by

$$S_{\text{Gauge}} = g_5 \int d^4x \int_{y_0}^{\infty} dy \sqrt{g} e^{-\Phi} \times [\bar{\Psi}_L e_A^M \gamma^A A_M \Psi_L + \bar{\Psi}_R e_A^M \gamma^A A_M \Psi_R].$$

The couplings of  $\psi_{\pm}^{(0)}$  to different KK gauge modes is then

$$g_{\pm}^n = g_5 \int_{y_0}^{\infty} dy f_A^{(n)} [(f_{L\pm}^{(0)})^2 + (f_{R\pm}^{(0)})^2]. \quad (8)$$

The dependence of the gauge couplings on the fermion locations  $c_{L,R}$  is shown in Fig. 2. It can be seen that for  $c_L > 1/2$  the couplings become universal. In the case where  $c_L = c_R$ , the couplings of one of the fermions would lie in the universal region the other would lie in the opposite part of the plot. However, with opposite Dirac masses,  $c_L = -c_R$ , we are able to place both fermions in a region of universal coupling at the same time and we would thus hope to suppress FCNC.

In hard-wall models the origin of the regions of universal couplings is quite clear and derives from the profile of the gauge field wave functions which are flat in the UV. Hence if fermion profiles are relatively UV localized, the gauge couplings will be universal. However, in the soft-wall model, looking at the profile of the zero mode of the massive gauge boson in Fig. 1(b), it is certainly not flat and one may wonder why we still find regions of universal gauge couplings. The explanation can be seen by considering the fermion profiles. Figure 3 shows the fermion wave functions contributing to the gauge coupling of  $\psi_+$  for  $c_L = -c_R = 0.7$  which are locations that live in an area of universal gauge couplings. While  $f_{L+}^{(0)}$  is heavily UV localized, we see that  $f_{R+}^{(0)}$  is actually peaked into the IR which we would expect to contribute to nonuniversal couplings. However, when one considers the relative size



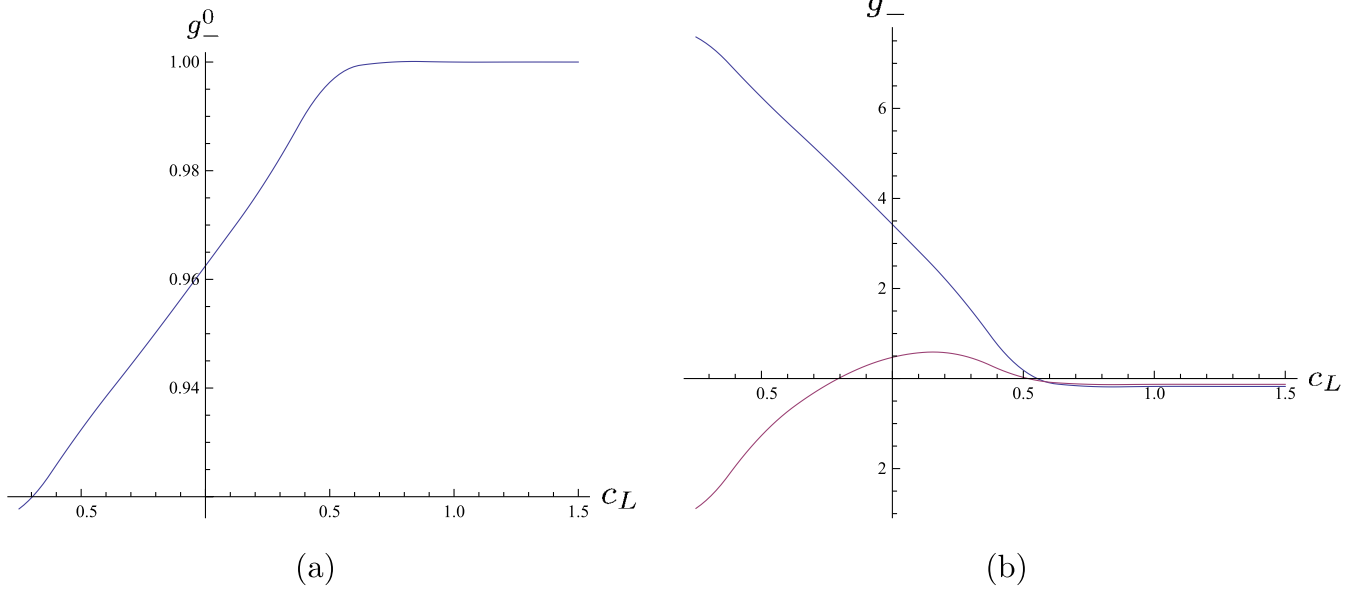


FIG. 2 (color online). Gauge couplings of the Z boson (a) and its first two KK states (b) as a function of the fermion location, normalized so that the coupling to the zero mode is unity for large  $c_L$ .  $k = 10^7$  TeV,  $\mu = 1$  TeV, and  $c_L = -c_R$ .

of the contributions of each of these wave functions to the gauge coupling as given by Eq. (8), we find  $\int (f_{R+}^{(0)})^2 / \int (f_{L+}^{(0)})^2 \sim 3 \times 10^{-4}$  i.e. almost the entire contribution to the gauge coupling comes from  $f_{L+}^{(0)}$  which is heavily UV localized. Although the relative dominance of  $f_{L+}^{(0)}$  is not that clear to see from Fig. 3, it becomes obvious when one realizes that it has a value at  $\mu y_0$  of about 2000. Also, its extreme UV localization can be seen by the fact that 99% of the area of  $(f_{L+}^{(0)})^2$  lies in the region  $\mu y < 0.01$ . Hence, the dominant contribution to the gauge coupling comes from a region in the extreme UV where the gauge profile is effectively given by its UV boundary value, thus producing universal couplings for fermions.

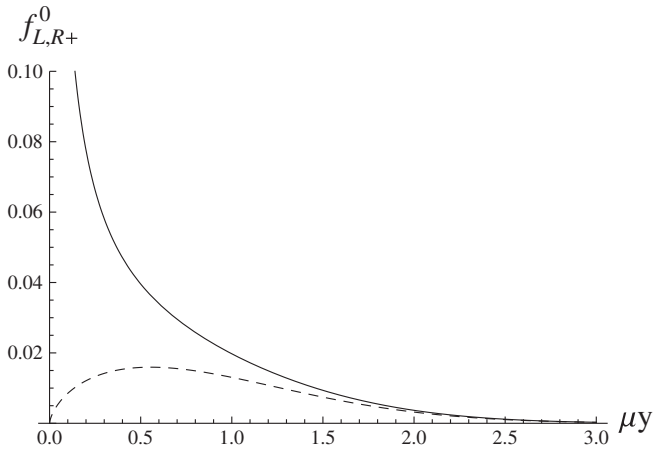


FIG. 3. Fermion zero modes with  $c_L = -c_R = 0.7$  for  $k = 10^7$  TeV and  $\mu = 1$  TeV.  $f_{L+}^{(0)}$  solid,  $f_{R+}^{(0)}$  dashed. Note that  $f_{L+}^{(0)}$  takes a value of around 2000 at  $y_0$ .

In order to generate the fermion mass hierarchy seen in the SM, we have to carefully choose the  $c$  parameters. The zero mode masses for different  $c$  parameters can be seen in Fig. 4. Unfortunately, the shape of the plot presents a problem for simultaneously generating a large hierarchy of masses and universal gauge couplings. While it is easy to generate a large hierarchy of masses for the choice of parameters  $c_L = c_R$ , as has been stated above, this is likely to lead to high rates of FCNC. In order to avoid these unacceptable rates, we would like both the fermions to reside in an area of universal gauge couplings; this corresponds to the top left corner of the contour plot where we have  $c_L > 1/2$  and  $c_R < -1/2$ . In this area the zero mode mass bottoms out at around  $\mu^2/k$  and it is not possible to create a large mass hierarchy. The solution to this is to simply increase the hierarchy of scales in the model. Keeping  $\mu = 1$  TeV, we can see from Fig. 5 that with  $k/\mu = 10^7$  and  $c_L = -c_R$ , the zero mode masses could cover the full range of charged lepton masses while remaining in an area of universal gauge couplings.

The reason for the zero mode mass having a minimum value for  $c_L = -c_R$  can be seen by considering the two different ways small masses are generated in such models. The zero mode masses are generated via Yukawa couplings which involve the overlap between  $\Psi_L$ ,  $\Psi_R$  and the Higgs VEV. In the case where  $c_L = c_R$ , the wave functions of the zero mode fermions become oppositely localized and can be arranged to have an arbitrarily small overlap with each other thus creating arbitrarily small masses, this is the mechanism used in the “split fermion” model [18]. However, when we take  $c_L = -c_R$  the fermions completely overlap each other and the zero mode mass is then entirely determined by their overlap with the Higgs.

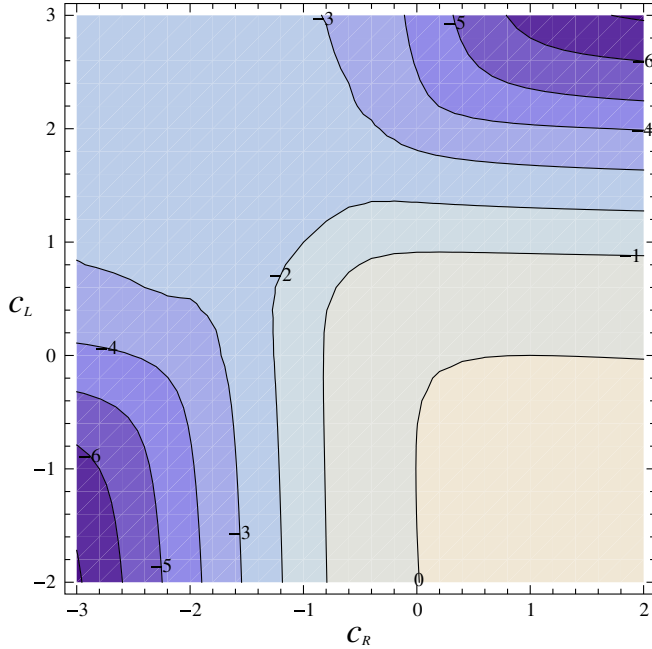


FIG. 4 (color online). Contour plot of  $\log_{10}(m_0/\mu)$  for the zero mode masses of fermions with  $\mu = 1$  TeV and  $k = 10^3$  TeV.

In RS models where the Higgs resides on the IR brane, this overlap can be made arbitrarily small by heavily localizing both the fermions at the UV brane. However, in the soft-wall model where the Higgs must necessarily propagate in the bulk and has a nonzero value at the UV brane, the fermion wave functions will always have a minimum overlap with the Higgs at  $y_0 = 1/k$  however much they are UV localized.

### B. Neutrino masses

We can generate small Dirac neutrino masses by introducing right-handed neutrinos and allowing Yukawa cou-

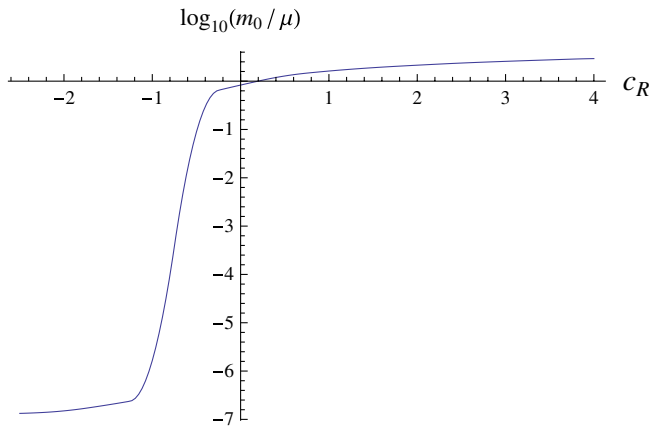


FIG. 5 (color online). Fermion zero mode masses with  $c_L = -c_R$ ,  $\mu = 1$  TeV and  $k = 10^7$  TeV.

plings between them, the Higgs and the left-handed neutrinos. A similar approach has been taken in the hard-wall case in Refs. [2,19]. The left-handed neutrinos will share the same  $c_L$  parameters as the corresponding left-handed charged leptons since they are part of the same doublet under  $SU(2)_L$ . We are then free to place the right-handed neutrinos in a suitable location in order to generate sub-eV masses. However, since we still require  $c_L > 1/2$ , we again find that we are unable to generate such small masses without vastly increasing the overall hierarchy in the model. It seems necessary therefore to work with a hierarchy similar to that proposed in the original Randall-Sundrum model. The issue of stabilizing such a large hierarchy is an important question, and it would be very interesting to redo our analysis in context of the stabilized model proposed in Ref. [13]. With this in mind, we choose  $k/\mu = 10^{15}$  and are able to produce neutrino masses of order 0.1 eV by choosing  $c_L = 0.6$  and  $c_R = -1.3$ .

### C. Three generations

When incorporating all three generations of leptons into our model, the Dirac mass terms  $M_L$  and  $M_R$  and the Yukawa coupling constants are promoted to  $3 \times 3$  matrices mixing the different generations. We assume that the basis of states in which the  $M_L$  and  $M_R$  are diagonal does not correspond to one in which the Yukawa couplings are diagonal. Rather than finding exact solutions for all three generations in such a scenario, our approach to this problem follows closely the method used in Ref. [5] for the Randall-Sundrum model. We solve the equations of motion individually for each generation with a Yukawa coupling  $\lambda_5 = 1$ , excluding fermion mixing, and use these solutions as basis from which we treat the full matrix of Yukawa couplings including mixings between the generations as perturbations. We specifically choose a large number of random Yukawa couplings, taking  $\frac{1}{2} < |\lambda_{5ij}| < 2$  with random sign,<sup>2</sup> and require that the average zero mode masses reproduce the observed lepton masses. We also choose to locate the left-handed fields of each generation close to each other in order to generate large neutrino mixings, in the spirit of Ref. [19]. However, we do not aim at reproducing the neutrino masses and mixings precisely. All we arrange for is an overall neutrino mass scale of order 0.1 eV. Using our tools, a full model of neutrino masses could be constructed. However, for the following estimate of lepton flavor violation, these details are not needed. Also, we use the fact that neutrino mixings are order unity.

In the case where we are not interested in generating neutrino masses via locations, we take only a moderate hierarchy of scales,  $k/\mu = 10^7$ . In this regime we choose the following three scenarios:

<sup>2</sup>We do not consider  $CP$  violation, i.e. we take  $\lambda_{5ij}$  to be real.

- (A):  $c_{L1} = 0.700$ ,  $c_{L2} = 0.700$ ,  $c_{L3} = 0.700$ ,  
 $c_{R1} = -1.376$ ,  $c_{R2} = -0.903$ ,  $c_{R3} = -0.703$ ,  
 (B):  $c_{L1} = 0.720$ ,  $c_{L2} = 0.700$ ,  $c_{L3} = 0.680$ ,  
 $c_{R1} = -1.373$ ,  $c_{R2} = -0.903$ ,  $c_{R3} = -0.704$ ,  
 (C):  $c_{L1} = 0.600$ ,  $c_{L2} = 0.600$ ,  $c_{L3} = 0.600$ ,  
 $c_{R1} = -1.430$ ,  $c_{R2} = -0.980$ ,  $c_{R3} = -0.790$

In the regime where we can also generate neutrino masses,  $k/\mu = 10^{15}$ , we choose

- (D):  $c_{L1} = 0.60$ ,  $c_{L2} = 0.60$ ,  $c_{L3} = 0.60$ ,  
 $c_{R1} = -0.82$ ,  $c_{R2} = -0.64$ ,  $c_{R3} = -0.55$ .

Our choices for the different scenarios (A), (B), and (C) are to demonstrate the effects of degenerate  $c_L$  localization (A), small separation in the  $c_L$  to introduce some nonuniversality in the left-handed sector (B), and placing the left-handed fermions closer to the IR brane (C). We expect the behavior to be quite general and thus only choose one scenario with a larger hierarchy. The mass of the first fermion KK states is about 1.5 TeV.

#### IV. FLAVOR VIOLATION

With a full three generations of leptons implemented as above, the transformation to fermion mass eigenstates will induce flavor violating couplings to gauge fields, in particular, the Z boson and its KK excitations<sup>3</sup>. We define the neutral current gauge couplings in the basis of mass eigenstates as

$$\mathcal{B}_{\pm}^{(n)} = \mathcal{U}_{\pm} \mathcal{G}_{\pm}^{(n)} \mathcal{U}_{\pm}^{\dagger},$$

where the unitary matrices  $\mathcal{U}_{\pm}$  diagonalize the full fermion mass matrices and  $\mathcal{G}_{\pm}^{(n)}$  are diagonal matrices that contain the couplings of the  $n$ th KK state of the Z boson to each fermion state as derived from Eq. (8) and normalized to the coupling of the muon (see also Ref. [5]). Flavor violation induced by these couplings is dependent on the

nonuniversality in the couplings of different flavor states and the mixing between the states. Different fermion locations increase the nonuniversality but at the same time lead to small mixing angles. Conversely, similar fermion locations produce large mixing but this is compensated by universal couplings.

As done in Ref. [5], we calculate the rates of the various flavor violating processes using the techniques developed for family nonuniversal Z' bosons [20]. The main difference being that there is no mixing between the different KK states of the Z boson, while the zero mode also has flavor violating couplings.

The first process we consider is the tree level exchange of a Z boson and its KK states mediating the process  $l_j \rightarrow l_i l_i \bar{l}_i$ . The rate for this process is given by [20]

$$\Gamma(l_j \rightarrow l_i l_i \bar{l}_i) = \frac{G_F^2 m_{l_j}^5}{48\pi^3} (2|C_{ij}^+|^2 + 2|C_{ij}^-|^2 + |D_{ij}^+|^2 + |D_{ij}^-|^2),$$

where

$$C_{ij}^{\pm} = \sum_n \frac{M_0^2}{M_n^2} (\mathcal{B}_{\pm}^{(n)})_{ij} (\mathcal{B}_{\pm}^{(n)})_{ii},$$

$$D_{ij}^{\pm} = \sum_n \frac{M_0^2}{M_n^2} (\mathcal{B}_{\pm}^{(n)})_{ij} (\mathcal{B}_{\pm}^{(n)})_{ii},$$

where we take the sum over the Z boson zero mode and the first two KK modes.  $M_n$  is the mass of the  $n$ th KK mode of the Z boson. Because of the Regge type behavior of the KK spectrum, it is not clear that the above series should converge. However, as noted in Ref. [16], due to the increasing IR localization of higher gauge boson KK modes, the couplings rapidly decrease and the series converges after only a few terms. In the fermion sector we take into account only the zero modes. Mixing with KK fermions is small, leading to negligible effects at the current precision.

The branching ratios for the above processes in the different scenarios we consider are then found to be

	(A)	(B)	(C)	(D)
$\text{Br}(\mu \rightarrow ee\bar{e}) :$	$2.7 \times 10^{-14}$	$5.1 \times 10^{-12}$	$4.6 \times 10^{-12}$	$2.5 \times 10^{-15}$
$\text{Br}(\tau \rightarrow \mu\mu\bar{\mu}) :$	$2.4 \times 10^{-14}$	$1.8 \times 10^{-12}$	$7.0 \times 10^{-13}$	$2.7 \times 10^{-12}$
$\text{Br}(\tau \rightarrow ee\bar{e}) :$	$2.6 \times 10^{-15}$	$1.5 \times 10^{-12}$	$6.6 \times 10^{-13}$	$2.8 \times 10^{-16}$

These numbers are obtained for a KK scale of 2 TeV. They are the result of averaging over random Yukawa couplings in the range stated above. The experimental

<sup>3</sup>Note that we work in a gauge field basis, where the Z boson zero mode is massive. Its properly weighted wave function is not flat, resulting in nonuniversal couplings to fermions. Alternatively, one could work in a basis, where the zero mode is massless. Then, only the KK states would couple nonuniversally.

bound  $\text{Br}(\mu \rightarrow ee\bar{e}) < 1.0 \times 10^{-12}$  [21] is satisfied in case (A) and case (D). However, it appears that the couplings are not universal enough to allow for much separation between the left-handed states (B), and placing the fermions too close to the IR brane (C) also exceeds the experimental bound. However, like in the hard-wall case, the rate for this process depends on the KK scale as  $1/M_{\text{KK}}^4$ . Thus with a KK scale of twice as big (i.e. take  $\mu = 2$  TeV while keeping  $k = 10^7$  TeV), scenario (B)

and scenario (C) would also acquire an acceptable rate. The experimental bounds for the other two processes  $\text{Br}(\tau \rightarrow \mu \mu \bar{\mu}) < 2.1 \times 10^{-8}$  and  $\text{Br}(\tau \rightarrow ee\bar{e}) < 2.7 \times 10^{-8}$  [22] are well satisfied in all the scenarios. Note that in Ref. [5] in a case similar to (A), (C), and (D), a branching ratio  $\text{Br}(\mu \rightarrow ee\bar{e}) = 5 \times 10^{-14}$  has been found for a KK scale of 10 TeV, translating into  $\text{Br}(\mu \rightarrow ee\bar{e}) = 3 \times 10^{-11}$  for a KK scale of 2 TeV. This demonstrates that lepton flavor violation is suppressed by up to four additional orders of magnitude in the soft-wall case.

We are also able to calculate the expected rate of  $\mu \rightarrow e$  conversion in a muonic atom. The most stringent bound comes from the Sindrum-II Collaboration [23] in  $^{48}_{22}\text{Ti}$  where  $\text{Br}(\mu^- N \rightarrow e^- N) < 6.1 \times 10^{-13}$ . We can calculate the branching ratio for this process by [20]

$$\text{Br}(\mu^- N \rightarrow e^- N) = \frac{G_F^2 \alpha^3 m_\mu^5}{2\pi^2 \Gamma_{\text{CAPT}}} \frac{Z_{\text{eff}}^4}{Z} |F_p|^2 (|B^-|^2 + |B^+|^2),$$

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(A)	(B)	(C)	(D)	
$\text{Br}(\mu^- N \rightarrow e^- N) :$	$1.6 \times 10^{-13}$	$3.3 \times 10^{-11}$	$2.7 \times 10^{-11}$	$1.4 \times 10^{-14}$

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Again, we find scenario (A) and scenario (D) lie within the experimental bounds but separating the states (B) or placing them too close to the IR brane (C) produces an unacceptable rate. Again, we find flavor violation suppressed with respect to the hard-wall case [5], making a KK scale of 2 TeV consistent with observations. However, next-generation experiments, such as PRISM at JPARC with a reach of  $\text{Br}(\mu^- N \rightarrow e^- N) \sim 10^{-16}$ – $10^{-18}$  could probe a KK scale of 6–20 TeV, i.e. the interesting parameter range of the present model.

A third set of processes considered in Ref. [20] are one-loop radiative lepton decays. Here the decay width is

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	(A)	(B)	(C)	(D)
$\text{Br}(\mu \rightarrow e\gamma) :$	$2.0 \times 10^{-17}$	$3.7 \times 10^{-15}$	$3.4 \times 10^{-15}$	$1.2 \times 10^{-18}$
$\text{Br}(\tau \rightarrow \mu\gamma) :$	$5.2 \times 10^{-16}$	$1.9 \times 10^{-14}$	$9.4 \times 10^{-15}$	$4.1 \times 10^{-14}$
$\text{Br}(\tau \rightarrow e\gamma) :$	$3.7 \times 10^{-17}$	$1.5 \times 10^{-14}$	$9.2 \times 10^{-15}$	$2.6 \times 10^{-18}$

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All of these branching ratios lie well within the experimental bounds  $\text{Br}(\mu \rightarrow e\gamma) = 1.2 \times 10^{-11}$  [25],  $\text{Br}(\tau \rightarrow \mu\gamma) = 4.4 \times 10^{-8}$ , and  $\text{Br}(\tau \rightarrow e\gamma) = 3.3 \times 10^{-8}$  [26]. Again these rates are suppressed relative to their hard-wall counterparts.

In scenario (D), processes such as  $\mu \rightarrow e\gamma$  can also be mediated by the KK states of the sterile neutrinos. This process was investigated for the Randall-Sundrum model in Ref. [27]. We use the formalism developed there and we find that in our model the branching ratio for this process is given by the relative coupling strength of the muon, the  $W$  and the KK muon neutrino to the zero mode muon neutrino times a loop factor. Because of the large mass differences

where

$$B^\pm = \sum_n \frac{M_0^2}{M_n^2} (\mathcal{B}_\pm^{(n)})_{12} [(2Z + N)(B_{u_L}^{(n)} + B_{u_R}^{(n)}) + (Z + 2N)(B_{d_L}^{(n)} + B_{d_R}^{(n)})],$$

and we take

$$B_{\psi_{L,R}}^{(0)} = g_{\psi_{L,R}}, \quad B_{\psi_{L,R}}^{(1)} = 0.19 g_{\psi_{L,R}}, \quad \text{and} \quad B_{\psi_{L,R}}^{(2)} = 0.14 g_{\psi_{L,R}}.$$

Here,  $g_{\psi_{L,R}}$  are the usual standard model quark couplings and we have taken the approximate values of the quark couplings to the higher KK gauge modes from the values derived in Ref. [16]. We also take  $Z_{\text{eff}} = 17.6$ ,  $F_p = 0.54$  and  $\Gamma_{\text{CAPT}} = 2.59 \times 10^6 \text{ s}^{-1}$  [24], where  $Z_{\text{eff}}$  is an effective atomic charge,  $F_p$  is a nuclear matrix element and  $\Gamma_{\text{CAPT}}$  is the muon capture rate.

We find branching ratios for the different scenarios of

$$\Gamma(l_j \rightarrow l_i \gamma) = \frac{\alpha G_F^2 m_{l_j}^3}{8\pi^4} (|\xi_+^{ij}|^2 + |\xi_-^{ij}|^2),$$

where the dipole moment couplings of an on-shell photon to the chiral lepton currents are given by

$$\xi_\pm^{ij} = \sum_n \frac{M_0^2}{M_n^2} (\mathcal{B}_\mp^{(n)} m_l \mathcal{B}_\pm^{(n)})_{ij},$$

where  $m_l$  is the charged lepton mass matrix. Using this, we obtain the following rates for radiative decays

of the sterile KK neutrinos, the Glashow-Iliopoulos-Maiani mechanism breaks down. We assume that the neutrino mixing angles are large, not leading to any suppression of the rate. Given a relative coupling of 0.0057, we find  $\text{Br}(\mu \rightarrow e\gamma) = 1.5 \times 10^{-13}$  which again lies within the experimental bounds for a KK scale of 2 TeV.

There are also contributions to  $\mu \rightarrow e\gamma$  related to the exchange of KK fermions [28,29], which were neither included in our estimate above nor in Ref. [27]. These contributions dominate the rate of radiative lepton decays in hard-wall models. A similar behavior is likely in the soft-wall model. However, given the suppressed rates for flavor violation in the latter, we expect that even including



these extra contributions, the rate for  $\mu \rightarrow e\gamma$  and a KK scale of 2 TeV should not exceed the experimental bound.

We have shown that lepton flavor violation can be suppressed in the soft-wall model and the amount of suppression depends on the fermion locations and the hierarchy of scales,  $k/\mu$ . The reason for this suppression comes from the ability to place the fermions in regions of universal gauge couplings i.e. heavily UV localized as explained in Sec. III A. When the hierarchy of scales in the model is increased by fixing  $\mu$  and increasing  $k$  the gauge boson profiles remain unchanged while the fermion profiles become even more UV localized thus providing an even greater suppression of flavor violation.

## V. CONCLUSIONS

In this paper we have studied the lepton sector of the SM in a soft-wall extra dimension, applying flavor dependent fermion locations to accommodate the observed lepton flavor structure. The Higgs is a bulk field, with a VEV that increases near the soft wall. We have, in particular, considered the inclusion of small Dirac neutrino masses and investigated the constraints on the model from lepton flavor violation mediated by the  $Z$  boson and its KK states. In order to do so, we first developed solutions for a massive gauge boson in the soft-wall background and found the profile is independent of the AdS curvature scale. In order to generate the masses of the charged leptons while keeping the fermions located in an area of almost universal gauge couplings, we find that we need to increase the hierarchy of scales in the model to around  $k/\mu = 10^7$ . When incorporating sub-eV neutrino masses we need a much larger hierarchy, and we choose  $k/\mu = 10^{15}$ , similar to the hierarchy between the Planck and the electroweak scales.

To incorporate three generations of leptons into our model, we solve the fermion equations of motion numerically, including an order one flavor diagonal Yukawa coupling and use these solutions as a basis of states from which we treat off-diagonal Yukawa couplings, connecting different generations, as perturbations. The mass term related to the diagonal Yukawa coupling is necessary to generate a normalizable wave function and cannot be treated as a perturbation. We can construct the full lepton mass matrices, including KK states and diagonalize them to find the fermion masses and mixings. However, to our level of precision, we can neglect the fermionic KK states. The locations of the left-handed fermions are dictated by the fact that we require large mixings in the neutrino sector. We take a large number of random Yukawa couplings and choose the locations of the right-handed fermions so that the averaged zero mode masses reproduce the SM charged lepton masses.

With the inclusion of off-diagonal Yukawa couplings, the transformation to mass eigenstates produces flavor violating couplings. We calculated the expected rates for various flavor changing processes for a number of different scenarios. We found that the soft-wall model is in fact mildly constrained when we consider a scenario with a low hierarchy of scales such as  $k/\mu = 10^7$ . The most stringent constraint comes from  $\mu \rightarrow e$  conversion in a muonic atom where we find that only the scenario where all the left-handed leptons have degenerate locations well toward the UV brane would occur at acceptable rates with a KK scale of 2 TeV. This is a considerable suppression of lepton flavor violation compared to hard-wall models, such as the one studied in Ref. [5]. Including a larger hierarchy of scales ( $k/\mu = 10^{15}$ ), it is also possible to generate sub-eV Dirac neutrino masses. In this case the model is even less constrained and most of the FCNC processes would occur at rates well below the experimental bounds. The most stringent bounds are coming from radiative decays, such as  $\mu \rightarrow e\gamma$ . Again, a KK scale of 2 TeV seems sufficient to keep the rate below the experimental bound. Our estimate for this rate does not include contributions from KK gauge bosons, and it would be interesting to include these in a more detailed analysis. Another obvious direction of research would be to extend the present setup to the quark sector, similar to an analysis that was performed recently in much detail for the hard-wall model in Ref. [30].

The soft-wall extra dimension continues to offer a valid model for electroweak physics, with constraints from precision data relaxed compared to the hard-wall model. Having said this, we have found that with a (gauge boson) KK scale of 2 TeV the complete lepton flavor structure can be accommodated while keeping rare processes below experimental bounds. In our setup the KK states of fermions have masses around 1.5 TeV, within reach of the LHC experiment. Thus the soft-wall framework seems to offer an alternative when it comes to suppressing flavor violation to models relying on flavor symmetries [29,31,32], a bulk Higgs [33], or to utilizing nonminimal representations under the  $SU(2)_R$  bulk gauge symmetry [34].

The parameter range with a large hierarchy  $k/\mu = 10^{15}$  is both attractive to further suppress flavor violation and necessary to accommodate neutrino masses. This rises the important question whether such a hierarchy can be stabilized, like in the way proposed in Ref. [13]. It would be very interesting to extend our analysis to such a framework.

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